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Specific Mental Arithmetic Difficulties and General Arithmetic Learning Difficulties: The Role of Phonological Working Memory

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Abstract

The aim of this paper was to examine the role of phonological working memory in specific mental arithmetic difficulties and general arithmetic learning difficulties (ALD; difficulties presenting in both mental arithmetic and written arithmetic). In Study 1, we categorized 53 sixth graders into a control group, a group with specific mental arithmetic difficulties, and a group with general ALD. The findings indicated the group with specific mental arithmetic difficulties performed significantly worse on the task involving phonological working memory than did the control group. However, a significant difference was not found between the group with general ALD and the control group. In Study 2 involving 54 sixth graders, we decreased the load of phonological working memory by changing the format of the problems from horizontal (more reliance on phonological codes) to vertical (more reliance on visual resources). We found that the group with specific mental arithmetic difficulties performed comparably to the control group. In other words, when the working memory load is reduced, they no longer lag significantly behind on mental arithmetic. However, the group with general ALD still performed significantly worse than the control group when the problems were presented vertically, indicating that reduced phonological working memory load did not alleviate their arithmetic difficulties. The findings in both studies suggested that poor phonological working memory might contribute to the underlying mechanism for specific mental arithmetic difficulties but not as much for general ALD.

Keywords

Phonological working memory, specific mental arithmetic difficulties, general arithmetic learning difficulties, elementary school students, mathematic learning

Introduction

The investigation of deficit in mental arithmetic has been a topic of interest in mathematical cognition, and there are two general strands of empirical studies. On one hand, some researchers have focused on the differences between individuals with and without mental arithmetic difficulties (D'Amico & Guarnera, 2005; Kuang, 2008; Wang et al., 2006). On the other hand, some researchers have considered that mental arithmetic difficulties are equivalent to arithmetic learning difficulties (ALD) and even considered them an essential component of mathematical learning disabilities (MLD; Barnes et al., 2006; Jordan & Montani, 1997; Rousselle & Noël, 2008), leading to studies that considered those with mental arithmetic difficulties and those with written arithmetic difficulties as a homogenous group (McLean & Hitch, 1999).

There are some mixed findings from the two strands of research. For example, students with specific mental arithmetic difficulties and those with general ALD demonstrated slower response rate and lower accuracy rate in mental arithmetic in comparison to same-age typical counterparts (McLean & Hitch, 1999; Wang et al., 2006). There are some mixed findings as well. For example, studies focusing on specific mental arithmetic difficulties reported that the phonological working memory is often compromised in this population (D'Amico & Guarnera, 2005; Kuang, 2008; Passolunghi & Siegel, 2001; Wang et al., 2006). Among studies that recruited students with both ALD and MLD, there were no clear conclusions about whether the phonological working memory was a weakness in ALD and MLD. For example, some researchers reported that individuals with ALD demonstrated deficits in phonological working memory and auditory processing (Geary & Brown, 1991), whereas some researchers reported null findings in individuals with ALD or MLD (McLean & Hitch, 1999; Van der Sluis et al., 2005). A plausible explanation of such mixed findings might be the fact that different research groups recruited individuals with different types of ALD. In short, it is reasonable to hypothesize that there might be two types of difficulties in arithmetic learning: specific mental arithmetic difficulties and general ALD. The purpose of this study was to explore the underlying differences between these two subtypes of difficulties in arithmetic in order to gain insights for future instructional intervention.

General ALD

A specific learning disorder in mathematics (or developmental dyscalculia) is a special case of persistent deficits in mathematics (Morsanyi et al., 2018). There has been a general agreement that there are at least two subtypes of children with mathematical difficulties, including one subtype that involves reading (verbal) deficits and the other that is only limited to nonverbal difficulties (Geary et al., 2000; Jordan & Hanich, 2003). The first subgroup is characterized by verbal deficiencies, reading, and spelling problems in the context of impaired mathematics problems (reading and spelling subgroup). The second subgroup is characterized by normal reading and spelling skills in the context of impaired performance in arithmetic (arithmetic group). ALD is a subcategory under learning difficulties (Micallef & Prior, 2004). ALD refers to specific difficulties in arithmetic that are not due to deficits in intelligence, reading, sensory processing, or educational deprivation and that occur in children, adolescents, and adults (Butterworth, 2005; Greiffenstein & Baker, 2002; Hitch & McAuley, 1991). Behavioral science provides the evidence that deficits in verbal and visual working memory (Geary, 2004, 2011; Swanson, 2011), inhibitory function (Blair & Razza, 2007; Swanson, 2011), ordering processing (Morsanyi et al., 2018), and attentional function (Ashkenazi et al., 2009; Swanson, 2011) might be contributing to the deficits in ALD. Based on such a definition, ALD consists of different types of difficulties in arithmetic, including difficulties in the areas of mental arithmetic or/and written arithmetic. However, the inclusion criteria were not consistent in the existing literature. For example, McLean and Hitch (1999) utilized performance in mental arithmetic and written arithmetic as the parameter to determine ALD. However, some researchers relied on performance only in mental arithmetic to determine individuals' arithmetic difficulties (Micallef & Prior, 2004), and it remained unclear whether these individuals had comorbid difficulties in written arithmetic.

Many studies on MLD reported that students with MLD often encountered difficulties in mental arithmetic (Rousselle & Noël, 2008), often selected less effective strategies than typically developing students and gifted students, were slower in mental arithmetic (Geary & Brown, 1991), demonstrated difficulties in direct retrieval of arithmetic facts (Geary et al., 2000; Jordan & Montani, 1997), were less accurate in multidigit mental arithmetic of subtraction, were slower and less accurate than typically developing students, retrieved fewer mathematic facts (Barnes et al., 2006), had difficulties in executing arithmetical procedures (Jordan et al., 2003), and were persistent in using immature problem-solving strategies (Geary et al., 2004; Temple & Sherwood, 2002). Some research reported that one of the core early-stage symptoms for Alzheimer's disease (AD) was difficulties in arithmetic (Parlato et al., 1992). Findings based on anatomical and functional magnetic resonance imaging images revealed that AD patients presented significant gray matter atrophy and reduced activations during encoding and recognition in the medial temporal lobes and inferior parietal/superior temporal associative areas (Rémy et al., 2004). In addition, patients with brain injury demonstrated deficits in retrieval of arithmetic facts (McCloskey et al., 1991). In practice, difficulty in mental arithmetic is often considered one of the criteria to define MLD.

Specific mental arithmetic difficulties

Passolunghi and Siegel (2001) reported that underachieving arithmeticians performed worse than the normally achieving group in immediate recall of numerical information, whereas such difference was not revealed when the materials included words. A plausible interpretation is that these children were slower in accessing the number presentations in long-term memory, leading to slower counting and lower digit span (Geary, 1993; Hitch & McAuley, 1991). D'Amico and Guarnera (2005) examined the working memory functions in children with poor arithmetic achievement but without reading disabilities, and the results concluded that underachieving arithmeticians performed poorly in the visual sketchpad task, central executive tasks, and phonological loop measure on the digit span forward. This evidence suggests that the digit span impairment experienced by underachieving arithmeticians indicates one of the symptoms of a general difficulty in acquiring long-term representation of numerical materials. Some individuals without a general and pervasive deficit in arithmetic demonstrate deficits in mental arithmetic. Is it plausible to postulate that there is a stand-alone subgroup of individuals who have particular difficulties in mental arithmetic but do not have substantial difficulties in other aspects of arithmetic?

There have been a handful of studies examining individuals with substantial difficulties in mental arithmetic, whereas their performance in written arithmetic was average and they did not have documented mental health disorders or intellectual disabilities (Kuang, 2008; Wang et al., 2006). Wang et al. (2006) utilized a set of addition problems through written format and oral format to identify individuals with difficulties in mental arithmetic (scores were two standard deviations below the mean) and without difficulties in written arithmetic (scores were within two standard deviations of the mean). In addition, they integrated teacher interviews and Raven's Colored Progressive Matrices (Raven et al., 1995) to rule out mental health disorders, intellectual disabilities, or learning difficulties due to lack of learning opportunities. Although Kuang (2008) did not provide an operational definition for specific mental arithmetic difficulties, his study implemented systematic screening measures to rule out participants with prior training in mental arithmetic, those with documented mental health disorders, those with extremely high or low IQ measured by Raven's Progressive Matrices, and those without significantly low scores in both mental arithmetic and written arithmetic. As a result, the participants with specific difficulties (scores were two standard deviations below the mean) in mental arithmetic and without the difficulties in the excluding criteria were considered as the group with specific mental arithmetic difficulties (Kuang, 2008).

Based on the classification and screening criteria in the existing literature, there are three types of testing conditions to identify specific mental arithmetic difficulties. First, some researchers considered the participants' performance in both mental arithmetic and written arithmetic and ruled out the possibility of documented intellectual disabilities or mental health disorders. Second, some researchers examined the participants' performance in both mental arithmetic and written arithmetic and categorized such difficulties as ALD. Third, some researchers only considered the participants' difficulties in mental arithmetic and considered such difficulties as a core symptom for MLD or ALD. However, it remained unclear whether these individuals might have comorbid difficulties in written arithmetic. In the present study, we considered the first and second types of testing conditions to categorize difficulties in arithmetic into two categories, including (a) those with difficulties in mental arithmetic but without difficulties in written arithmetic (termed specific mental arithmetic difficulties) and (b) those with difficulties in both mental arithmetic and written arithmetic (termed general ALD). Thevenot and Castel (2012) reported that training in mental arithmetic enhanced performance in mental arithmetic but not in written arithmetic, whereas training in written arithmetic enhanced performance in both mental arithmetic and written arithmetic. It is reasonable to hypothesize that the underlying mechanism for specific mental arithmetic difficulties might differ from the mechanism for general ALD.

Differences in mental arithmetic and written arithmetic

To understand the underlying mechanism for mental arithmetic and written arithmetic, it is essential to compare the differences in the cognitive process involved in the two types of arithmetic operations. Mental arithmetic (or mental calculation) refers to a cognitive process that does not rely on external tools such as calculator, pencil and paper, or computer and in which the individual relies on mental problem-solving and retrieval of arithmetic facts to execute the arithmetic problems and directly provide the answers (Zhang et al., 2002). According to the theory of information processing, mental arithmetic warrants three stages in cognitive processing: encoding, calculation, and response (Campbell & Epp, 2005; Dehaene, 1992; DeStefano & LeFevre, 2004; McCloskey, 1992).

During the encoding stage, the individual encodes the stimulus through auditory, semantic, or visual modalities and temporarily holds the encoded information in memory. When numeric values ($8+7=?$) are presented through number words instead of digits, individuals' performance is remarkably worse on the measures of response time and accuracy (eight plus seven equals what?); this is the format effect during the encoding stage (Dehaene & Cohen, 1995; Noël et al., 1997). Researchers have explained that participants might be far more familiar with arithmetic problems presented through the modality of digits than through the modality of number words (Schunn et al., 1996).

During the calculation stage, the individuals retrieve arithmetic facts (e.g., $2+2=4$) from long-term memory, execute the calculation procedure (e.g., $9+3=10+3–1+12$), and sometimes perform carrying and borrowing. The problem size effect occurs at this stage. When the numeric values are larger, the response time is longer and accuracy rate is lower (e.g., $1+1$ is easier than $8+9$; Ashcraft, 1992; Campbell et al., 2004). Researchers postulated that individuals might encounter more difficulties retrieving arithmetic facts with larger values and tend to use more complex operation procedures in situations with larger numeric values (Campbell & Alberts, 2009; Campbell & Xue, 2001; Zbrodoff & Logan, 2005).

During the response stage, the individuals need to respond through different modalities depending on the tasks and the testing conditions (e.g., oral response or written response). Successful execution of mental arithmetic problems relies on fluent and successful coordination of all three stages of operation. In terms of the relations between mental arithmetic and written arithmetic at the stages of encoding, calculation, and response, some researchers considered them as two independent processes (Frampton & Faulkenberry, 2016; Zhou, 2011), whereas some researchers considered them as two intercorrelated processes (Campbell & Alberts, 2009; Campbell & Epp, 2005; Metcalfe & Campbell, 2008).

Similarly, written arithmetic also requires the stages of encoding, calculation, and response. However, the procedures involved in written arithmetic and mental arithmetic might differ to some degree (Thevenot & Castel, 2012). For example, most written arithmetic problems are presented through a visual modality (e.g., on paper, on chalkboard), whereas mental arithmetic problems are often presented through an auditory modality. During written arithmetic, individuals retrieve relevant arithmetic facts and then transfer the arithmetic facts into written format, instead of temporarily holding the intermediate results in working memory. For written arithmetic, individuals provide answers in written format, whereas individuals often respond orally during mental arithmetic.

The role of phonological working memory in mental arithmetic

Working memory plays an important role in mathematics problem-solving because many mathematics tasks involve the activation of working memory (Swanson & Jerman, 2006). According to Peng et al. (2016), there are three moderators that could explain the variance in mathematics explained by working memory, including domain of working memory, mathematics skills (specific mathematic domain knowledge such as basic number knowledge, whole-number calculation, word-problem solving), and sample type (different participant population). From a theoretical perspective, working memory as a contributing factor to mathematics performance could be domain-general or domain-specific (Miyake & Shah, 1999). According to the domain-specific theory, the operation of working memory depends on domain knowledge and is largely affected by domain specificity (Ericsson & Kintsch, 1995). Thus, verbal working memory and numerical working memory may be strongly related to number-related mathematical tasks such as calculation, whereas visuospatial working memory may be strongly related to mathematics tasks with a strong visuospatial component such as geometry. The domain-general model of working memory postulates that the relation between working memory and mathematics should be invariant whether working memory is measured by using verbal, visuospatial, or numerical materials. An example of the domain-general working memory model is proposed by Baddeley (1986). According to Baddeley and Hitch (1974), working memory consists of the central executive and two subcomponents: the articulatory (later phonological) loop and the visuospatial scratchpad (later sketchpad; Baddeley, 1992). The phonological loop, also known as phonological working memory, comprises a phonological input store and an articulatory rehearsal process (Baddeley, 2003). It is well known that the phonological input store can only hold memory traces for a few seconds and will quickly fade out, unless the articulatory rehearsal process is activated to refresh the information (Baddeley & Hitch, 1974; Baddeley & Logie, 1999).

Performance on simple arithmetic tasks varies according to the presentation modalities associated with the problems, partially due to the different types of working memory that are required by the presentation modalities. During written arithmetic, the calculation procedure places fewer demands on phonological working memory because intermediate results can be stored through external media, such as paper or whiteboard (Trbovich & LeFevre, 2003). During mental arithmetic, the calculation procedure and temporary holding of intermediate results in mind all place demands on phonological working memory (Cavdaroglu & Knops, 2016; Imbo & LeFevre, 2010; Lee & Kang, 2002). Levine et al. (1992) examined children who were 4- to 6-year-olds on problems presented nonverbally and problems presented verbally. Children in the 4-year-old group performed better on nonverbal problems (using disks) than on verbal problems (using story problems and number-fact problems), whereas such difference did not exist in the 6-year-old group. Huttenlocher et al. (1994) suggested that young children might be using a mental model (Halford, 1993; Johnson-Laird, 1983), which might work for nonverbal arithmetic problems. However, verbal problems may require additional quantitative skills, symbols, and translation of information across modalities (from verbal to nonverbal), and such processes might be difficult to be achieved at a young age. Rasmussen and Bisanz (2005) examined the involvement of subcomponents of working memory in early arithmetic development. The results concurred with the previous findings that preschool children performed better on nonverbal problems than on verbal problems, and the visual-spatial working memory was the only unique predictor of performance on nonverbal problems. Among Grade 1 children, they performed comparably on nonverbal and verbal problems, and phonological working memory was the best predictor on performance on verbal problems. Similarly, in Grade 4 and Grade 5 students, students' accuracy and response time on multiplication facts were most susceptible to phonological loop influence, suggesting that multiplication facts are stored in a verbal modality, and the activation of such arithmetic facts relies more on verbal modality in older elementary children (Liu et al., 2017).

In order to accomplish mental arithmetic problems, there are several steps that might be expected. At the stage of encoding and storing procedures, the problems presented through auditory modality often activate phonological working memory. For example, when problems were presented as digits (“8 + 7 = ?”) versus number words (“eight plus seven equal what?”), individuals performed worse on response time and accuracy in the latter situation (Blankenberger, 2001; Campbell & Fugelsang, 2001). When the problems were presented through an auditory modality, extra phonological working memory load had a larger effect on mental arithmetic; when the problems were presented through a written modality, extra visuospatial sketchpad load rendered a larger effect on mental arithmetic (Heathcote, 1994; Logie et al., 1994). Extra phonological loop load had a larger effect on mental arithmetic when the problems were presented in a horizontal format than in a vertical format (Trbovich & LeFevre, 2003). When the numbers were made visible for a short period of time, extra phonological working memory load generated a large effect on mental arithmetic; when the numbers were made visible throughout the calculation, extra phonological working memory load had no effect on mental arithmetic (Furst & Hitch, 2000).

Phonological working memory is activated during the calculation stage. In a study using one-digit mental arithmetic, phonological working memory was activated to retrieve arithmetic facts corresponding to the presented problems from long-term memory (Kaye et al., 1989). However, another study reported that central executive played a larger role in one-digit mental arithmetic than did phonological loop (Ashcraft et al., 1992). Phonological loop significantly affected performance in multiplication, but not in subtraction (Lee & Kang, 2002). Phonological loop rendered a larger effect on problems with a large minuend (i.e., minuend was larger than 11) than it did on problems with a smaller minuend (i.e., minuend was smaller than 9; Seyler et al., 2003). Phonological loop rendered a nonsignificant effect when the participants utilized direct retrieval and yielded a significant effect when the participants utilized more complex strategies such as decomposition (Hecht, 2002). It appears that the role of phonological loop is dependent on the languages used for problem-solving. For example, extra load on phonological loop did not have an impact on mental arithmetic in participants who spoke English and German (De Rammelaere et al., 1999, 2001). However, extra load on phonological loop rendered a significant impact on the performance of mental arithmetic in participants who spoke Korean or French (Lee & Kang, 2002; Lemaire, 1996). It is plausible that Korean-speaking and French-speaking participants were taught to use auditory approaches to become fluent with arithmetic facts (Liu et al., 2017). Phonological loop plays an important role in multidigit mental arithmetic. Phonological loop is involved in retrieval of arithmetic facts, maintaining of intermediate results, and carrying (Heathcote, 1994; Seitz & Schumann-Hengsteler, 2002).

During the stage of response, most problems of mental arithmetic require the participants to respond through the oral modality, which obviously involves phonological working memory (DeStefano & LeFevre, 2004). However, there is very sparse research focusing on the role of phonological working memory during the stage of response. In short, most previous studies utilized dual tasks by increasing the load of phonological working memory through the secondary task. There are few studies that alternated the load of phonological working memory through the primary task (i.e., the mental arithmetic task itself). In particular, there are no studies that examined the role of phonological working memory by decreasing the load on phonological working memory. Many studies focused on adult learners, with little attention to young learners who have difficulties in mental arithmetic. Many studies did not differentiate learners who have specific mental arithmetic difficulties from those who have general ALD.

The role of phonological working memory in specific mental arithmetic difficulties

Individual differences in phonological working memory have an impact on learners' performance in mental arithmetic. On one hand, many studies suggested that the capacity of phonological working memory significantly predicted individuals' performance in mental arithmetic, with stronger phonological working memory predicting better performance in mental arithmetic (Adams & Hitch, 1997; Berg, 2008; Gao, 2011). On the other hand, some studies did not show such a relation (Fuchs et al., 2006). In Chinese second through fourth graders, those with difficulties in mental arithmetic demonstrated weaker phonological working memory than the control group (Wang et al., 2006). In Chinese second graders, students with mental arithmetic difficulties demonstrated weaker performance in digit span than typical students (Kuang, 2008).

Individuals with ALD might have a wide range of cognitive difficulties. For example, there might be deficits in long-term retrieval of linguistic or nonlinguistic information (D'Amico & Passolunghi, 2009), visual-spatial skills (Geary, 2003; Rourke, 1993), and auditory processing (Hecht et al., 2001). Many studies focusing on students with learning disabilities found that these individuals often demonstrated deficits in central executive and visuospatial sketchpad (D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Mammarella, 2010; Passolunghi & Siegel, 2001), and some reported deficits in phonological loop (Geary et al., 2004). However, other studies reported null findings for difficulties in phonological loop (McLean & Hitch, 1999; Van der Sluis et al., 2005).

In short, research focusing on specific mental arithmetic difficulties suggested that individuals with stand-alone difficulties in mental arithmetic demonstrated deficits in phonological working memory. However, when researchers recruited different types of arithmetic difficulties, such as specific mental arithmetic difficulties and general ALD, and considered them as a homogeneous group, the findings were mixed. It appears to be clinically meaningful to separate participants with specific mental arithmetic difficulties from those with general ALD.

The purpose of the study

The present study classified students with arithmetic difficulties into two groups: those with specific mental arithmetic difficulties and those with general ALD (i.e., difficulties in both mental arithmetic and written arithmetic). Students with specific mental arithmetic difficulties did not present salient problems in arithmetic presented on paper, which indicated that they did not have substantial difficulties with arithmetic operations. Students with general ALD presented substantial problems in arithmetic presented on paper or through oral modalities, which suggested that they had persistent and chronic difficulties with basic arithmetic operations and that such operation difficulties might hinder their performance in mental arithmetic as well. Thus, we examined how phonological working memory might be a factor contributing to specific mental arithmetic difficulties but not to general ALD, through the perspective of individual perspective and task perspective.

In Study 1, we examined the phonological working memory of participants through the perspective of individual differences. We compared the capacities of phonological working memory in students with specific mental arithmetic difficulties, students with general arithmetic difficulties, and students in a control group.

Hypothesis 1: We hypothesized that students with specific mental arithmetic difficulties would demonstrate weaker phonological working memory than students with general ALD and students in the control group, whereas phonological working memory weakness might not be as substantial in the group with general ALD.

In Study 2, we examined the role of phonological working memory through the perspective of task differences. Different from the dual task used in many previous studies, we alternated the load of phonological working memory through the primary task.

Hypothesis 2: We hypothesized that tasks with lower demands on phonological working memory would help students with specific mental arithmetic difficulties to perform better on mental arithmetic problems, resulting in nonsignificant differences between this group and the control group. We hypothesized that tasks with lower demands on phonological working memory might not help students with general ALD improve performance in mental arithmetic problems.

If both Hypotheses 1 and 2 held true, we could prove that phonological working memory has an impact on specific mental arithmetic difficulties from the perspective of individual differences and task differences.

Study 1

Design and participants

In Study 1, we initially recruited 183 sixth graders in China (i.e., Chinese elementary school consists of first through sixth grades). Parent consent and child assent forms were distributed. Demographic information sheets were collected before the test administration, and individuals with documented mental health disorders and intellectual disabilities were excluded from the study. Only 177 participated in mental arithmetic examinations, and 170 participated in both mental and written arithmetic examinations. The cutoff scores for screening children with mathematics learning difficulties are variable in different empirical studies (e.g., 35th percentile in Geary et al., 2000; Hanich et al., 2001; 30th percentile in Geary et al., 1999, 2004; 25th percentile in Jordan et al., 2003). Based on Kuang (2008) and Wang et al. (2006), we defined students with scores of mental arithmetic below the 25th percentile rank (corresponding to a raw score of 8) and with scores of written arithmetic above the 75th percentile rank (corresponding to a raw score of 24) as those with specific mental arithmetic difficulties. We defined students with scores of mental arithmetic below the 25th percentile rank and scores of written arithmetic below the 25th percentile rank as those with general ALD. We defined students with scores for both mental arithmetic and written arithmetic above the 75th percentile as those without any arithmetic difficulties (the control group). In total, we identified 22 students with specific mental arithmetic difficulties, 23 with general ALD, and 107 without any difficulties.

Because there were far more students meeting the criteria for the control group, we randomly selected some of them to participate in the study. Because of sickness, unexpected absence during testing days, and other reasons, final enrollment in the study was 53 participants (male = 29, female = 24), including 12 students with specific mental arithmetic difficulties, 20 with general ALD, and 21 in the control group. The mean of age was 11.8 years old, with the range of 11 years 4 months to 12 years 3 months.

Measures and procedures

Arithmetic problems. Because Chinese curriculum for elementary math stresses practice and drills, children are very fluent with addition and subtraction. If we used addition and subtraction, the variance within the participants would be very small. Thus, we first designed 40 items for mental multiplication and then converted the 40 items of mental multiplication problems to 40 division problems. We followed the rules that (a) there were no zeroes in the ones place, (b) none of the multiplicands or multiplicators were one, and (c) each number had a different numeric value in each digit (e.g., 33×7 was not allowed). As a result, we generated 80 items for mental multiplication (i.e., two-digit × one-digit) and mental division (i.e., two-digit ÷ one-digit or three-digit ÷ one-digit). We designed 32 written arithmetic problems, of which half were multiplication problems (i.e., two-digit × two-digit) and the other half were division problems (i.e., three-digit ÷ two-digit or four-digit ÷ two-digit).

For mental arithmetic problems, participants were directed to take the test in a computer room. All items were presented on programmed computers using Inquisit 4 software. All questions were presented in black print against a white background using Times New Roman 60-point font. For each question, the participants were instructed to enter the answer using the keyboard. After the participants entered their response, the answer was presented after the symbol of "=." Shortly after the participants pushed the "enter" button, the next problem was presented. The mental arithmetic problems were presented through group administration. The examiner read the examination directions in the computer room. Each participant had an opportunity to work on six practice items. After the practice items, if the participant understood the examination rules, then he/she could start the examination. If a participant needed more practice, he/she could work on practice items again until the rules of the examination were fully understood. To avoid an order effect, all problems were randomly presented. All of the participants had prior experience with taking tests on computers. Thus, there were no concerns about the participants' lack of experience on computerized tests.

For written arithmetic problems, we conducted the test through group administration. The examiner read the examination directions to the group. The participants were directed to work on the written arithmetic problems on the worksheets provided to them. All items were printed in black ink using Times New Roman 12-point font on the front and back sides of letter-size paper. On each side of the page, there were four rows of questions, and each row had four questions. Each participant had 10 minutes to finish the questions. Each participant was instructed to write the response on the space adjacent to each question. Figure 1 shows the sample questions that were horizontally presented.

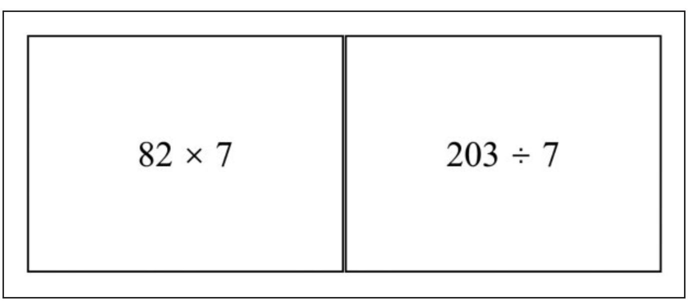

Figure 1. Examples of horizontally presented problems (Study 1).

To counterbalance practice effects, half of the participants completed the mental arithmetic problems first and then completed the written arithmetic problems. The other half of the participants completed the examinations in the reverse order. The internal consistency for the testing items was 0.72.

Phonological working memory task. Phonological working memory task based on digits is one of the most frequently utilized tools and has been validated in many previous studies (Chen & Wang, 2006; Li et al., 2003; Salthouse & Babcock, 1991; Turner & Engle, 1989; Wu, 2010; Xing et al., 2016). It required the participants to solve simple arithmetic problems (one-digit addition and subtraction). The examiner explained the purpose of and directions for the test before each participant started the testing. Prior to the start of testing, a stimulus "+" appeared on the screen for 1000 ms before the first test item was presented for 1500 ms. All questions were presented in black print against a white background using Times New Roman 50-point font. In the center of a computer screen used for testing purposes, simple arithmetic problems (e.g., $1 + 1 = ?$, $1 + 2 = ?$, 6–1+?) were presented. Then, the participants did mental calculations and entered all answers (e.g., 2, 3, 5), respectively. The question series started with two items in a row. If the participant entered correct answers on the first two items, the computer presented three-item, four-item, and five-item trials in a row, and so on. For each level (e.g., two-item level, three-item level), there were three trials. If the participant failed two out of three trials, the test was automatically discontinued. The internal consistency for this measure was 0.85.

Keyboard typing task. Ideally, the participant provides an oral response to the examiner when a mental arithmetic problem is presented. However, one-on-one oral responses to the examiner require individualized test administration for all participants, which might substantially interrupt students' daily routine at school. We adopted a group administration approach, so all students could enter the responses on computers situated in the computer lab. It remained unclear whether each participant had comparable typing speeds when they entered the responses. Thus, we controlled the speed of typing as a covariate, so students would not be penalized for slow typing speed. After the participants completed all tasks, they were directed to complete a typing task on the designated computer. The computer randomly generated 20 number strings. The participants were instructed to enter the exact number string they saw each time the item was presented. All number strings had two digits, and the numbers in the tens place and the ones place were different. The computer automatically recorded the accuracy and response time for each item. The internal consistency for this measure was 0.79.

Data analysis and results

To statistically control for typing speed, we used the participants' typing speed on all correctly answered items and calculated their typing speed on items that were correctly answered (M = 2436.34, SD = 1015.82). We excluded outliers that were three standard deviations above the mean. In the subsequent analysis, we controlled the typing speed as a covariate.

In total, we had 53 participants, including 12 students with specific mental arithmetic difficulties, 20 students with general ALD, and 21 students in the control group. We controlled the speed of typing as a covariate and ran analysis of covariance (ANCOVA) to evaluate the means of mental arithmetic (range from 3 to 56 points) and written arithmetic (range from 0 to 24) across the groups. The results showed that there were significant mean differences on the measure of mental arithmetic ($F(2, 54) = 60.01$, $p < .001$, $\eta^2_{partial} = .690$) and the measure of written arithmetic ($F(2, 54) = 38.57$, $p < .001$, $\eta^2_{partial} = .588$) among the three groups. We used Bonferroni test to examine the between-group difference. On the measure of mental arithmetic, the control group (M = 39.57, SD = 7.72) performed significantly better than the group with specific mental arithmetic difficulties (M = 18.41, SD = 4.94, $p < .001$, 95% CI [14.41, 25.51]) and the group with general ALD (M = 15.05, SD = 5.67, $p < .001$, 95% CI [17.59, 28.65]). There was no significant difference between the group with specific mental arithmetic difficulties and the group with general ALD ($p > .05$, 95% CI [–1.97, 8.29]). Similarly, on the measure of written arithmetic, the control group (M = 14.76, SD = 4.18) performed significantly better than the group with specific mental arithmetic difficulties (M = 11.12, SD = 3.06, p = .02, 95% CI [0.39, 6.13]). The range of written arithmetic for the group with specific mental arithmetic difficulties was [8, 18], which was within two standard deviations of the range of written arithmetic for the control group. In addition, on the measure of written arithmetic, the control group outperformed the group with general ALD (M = 4.55, SD = 2.01, $p < .001$, 95% CI [6.91, 12.63]). On the measure of written arithmetic, the group with specific mental arithmetic difficulties also outperformed the group with general ALD ($p < .001$, 95% CI [3.85, 9.16]). According to Wang et al. (2006), our classification of three groups was valid.

In terms of the phonological working memory task, we used "n–1" as the span of the participant's phonological working memory. For example, if the computer program discontinued the phonological loop task at the level of four items, this indicated that the participant failed at that level (four items in a row). Thus, the participants were successful at the level of three items in a row. Then, a point of "3" was recorded for that participant on the phonological loop task.

As shown in Figure 2, the control group had the highest scores on the phonological working memory task (M = 6.14, SD = 1.46), followed by the group with general ALD (M = 4.60, SD = 2.16), and the group with specific mental

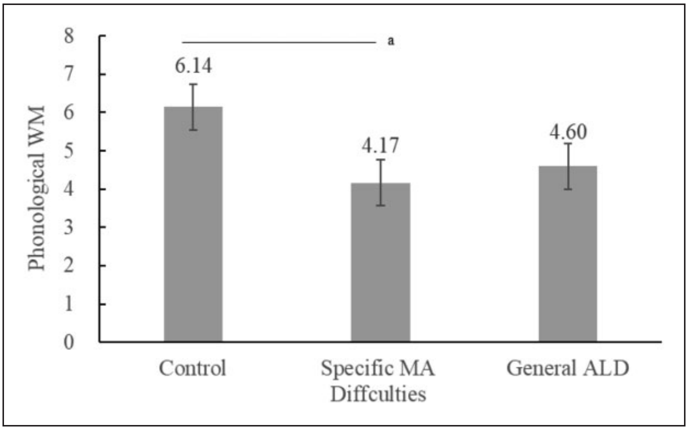

Figure 2. Phonological working memory in three groups (Study 1).
MA: mental arithmetic; ALD: arithmetic learning difficulties.
[a]Marginal significance.

arithmetic difficulties had the lowest scores (M = 4.17, SD = 1.70). All participants' scores were ranged from 1 to 9. To compare between-group mean differences on the measure of phonological working memory, we had the phonological working memory task as the dependent variable and the typing speed as the covariate, and we conducted an ANCOVA. Three groups had marginally significant differences on the means ($F(2, 49) = 2.94$, p = .062, $\eta^2_{partial} = .107$). We used Bonferroni test to examine the between-group difference. Further analysis revealed that the control group (M = 6.14, SD = 1.46) performed significantly better than the group with specific mental arithmetic difficulties (M = 4.17, SD = 1.70, p = .058, 95% CI [– .04, 3.23]). A significant difference was not revealed between the control group and the group with general ALD (M = 4.60, SD = 2.16, $p > .05$, 95% CI [– .72, 2.46]). A significant difference was not revealed between the group with specific mental arithmetic difficulties and the group with general ALD ($p > .05$, 95% CI [– 2.36, .90]).

Discussion

The findings generally supported our H1 that students with specific mental arithmetic difficulties would demonstrate weaker phonological working memory than the control group, whereas the group with general ALD might not show a significant deficit on a phonological working memory task. Results in previous studies indicated that difficulties in phonological working memory were often demonstrated by students with mental arithmetic difficulties (D'Amico & Guarnera, 2005; Kuang, 2008; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Wang et al., 2006; Zhou et al., 2006). Our findings revealed that both the group with specific mental arithmetic difficulties and the group with general ALD had lower scores on phonological working memory in comparison to the control group. However, only the group with specific mental arithmetic difficulties performed significantly worse than the control group, whereas a significant difference was not revealed between the group with ALD and the control group. The results explained the mixed findings in previous studies, which might have included both students with specific mental arithmetic difficulties and students with general ALD, leading to null findings of deficits in phonological working memory.

Students with specific mental arithmetic difficulties demonstrated difficulties only in mental arithmetic but not in written arithmetic. It is plausible that operation difficulties occur in the process that distinguishes mental arithmetic from written arithmetic. In the present study, we controlled design variables such as continuing presentation of the problems and requesting responses through visual modalities in both mental arithmetic and written arithmetic. We hypothesized that the difficulties occurred during the calculation process, not in the process to retrieve arithmetic facts. For the calculations during written arithmetic, the process was externalized by the assistance of pencils and paper, and many intermediate results and operation steps could be completed on paper, which largely decreased the demand on phonological working memory. However, the calculations during mental arithmetic relied on large involvement of phonological working memory to temporarily hold arithmetic facts and maintain intermediate results while the participants were attempting to problem-solve. During the testing, we observed that some participants demonstrated behaviors such as finger counting or finger movement during mental arithmetic to assist problem-solving, whereas such behaviors were not observed during written arithmetic. Thus, it is plausible to conclude that students with specific mental arithmetic difficulties had substantial difficulties with phonological working memory, which might contribute to their poor performance during mental arithmetic. However, students with general ALD might possess other types of difficulties, such as poor basic calculation skills, which contribute to poor performance in calculation in general regardless of whether they had or did not have poor phonological working memory. In the present study, the group with general ALD and the control group did not show a significant difference on phonological working memory.

Based on the number of students with specific mental arithmetic and students with general ALD during our initial recruitment, the occurrence of these two types of arithmetic difficulties was substantial. Because the cognitive processes involved in mental arithmetic and written arithmetic are qualitatively different, the differentiated grouping for students with specific mental arithmetic and students with general ALD has both theoretical and practical values.

Our phonological working memory task was designed by using simple arithmetic problems as the stimuli. Thus, a question might be raised regarding whether it actually measured students' arithmetic problem-solving instead of phonological working memory. We had two justifications for this design. First, all of the items used for the phonological working memory task were single-digit addition or subtraction problems, for which most Chinese sixth graders should achieve a high level of automaticity because Chinese mathematic curriculum emphasizes drills and practice on arithmetic facts. Thus, it is unlikely that the performance difference on phonological working memory task was due to their skills on single-digit addition or subtraction, which should be mastered by the end of first grade in Chinese schools. Second, although students with general ALD had lower performance on the mental arithmetic task, their performance on the phonological working memory task was not significantly different from the control group. Thus, we considered that the phonological working memory task used in the present study was valid.

Study 2

In Study 1, we found evidence of poor phonological working memory performance in students with specific mental arithmetic difficulties through the examination of individual differences. In Study 2, we alternated the load on phonological working memory through the perspective of task characteristics. We hypothesized (H2) that tasks with lower demands on phonological working memory would help students with specific mental arithmetic difficulties perform better on mental arithmetic problems, but it would not help students with general ALD.

Participants

The recruiting procedure was similar to Study 1. In total, we had 54 participants (male = 29, female = 25): 20 students in the control group, 14 students with specific mental arithmetic difficulties, and 20 students with general ALD. The mean of the age was 11.75 years old, with the range of 11 years 5 months to 12 years 2 months. We went to the same participating school to recruit the participants; however, there was a two-month interval between Study 1 and Study 2. It remained unclear how many new mathematics skills the students learned in two months. Thus, we treated the participants in Study 2 as an independent sample.

Measures and procedures

Arithmetic problems. We designed 80 items for the mental arithmetic task. The design of the task, the number of multiplication and division problems, and the level of difficulty were similar to the mental arithmetic problems we used in Study 1. To avoid practice and memorization effects, the problems in Study 2 were not identical to those used in Study 1.

To avoid an order effect, all problems were randomly presented using E-prime software. All items were presented in black type using Times New Roman 60-point font. All mental arithmetic problems in Study 1 were presented in a horizontal format, whereas all mental arithmetic problems in Study 2 were presented in a vertical format, which was reported to place less demand on phonological working memory (Trbovich & LeFevre, 2003). The internal consistency for the testing items is 0.72. Figure 3 shows examples of one multiplication problem and one division problem. All procedure rules were similar to those in Study 1.

Keyboard typing task. We used the same keyboard typing task that was presented in Study 1 and considered it a covariate for further analysis. The internal consistency for this measure was 0.79.

Data analysis and results

To compare the between-group mean difference (see Figure 4) on the measure of vertically presented mental arithmetic, we had the scores of mental arithmetic as the independent variable and the typing speed as the covariate, and we conducted an ANCOVA. Three groups showed significant differences on the means ($F(2,50) = 12.45$, $p < .001$, $\eta^2_{\text{partial}} = .332$). Further Bonferroni test revealed that the control group (M = 46.00, SD = 11.23) performed significantly better than the group with general ALD (M = 25.40, SD = 9.69, $p < .001$, 95% CI [8.29, 26.11]), but a significant difference was not revealed between the control group and the group with specific mental arithmetic difficulties (M = 37.86, SD = 9.61, $p > .05$, 95% CI [–3.94, 14.67]). The group with specific mental arithmetic difficulties performed better than the group with general ALD ($p < .01$, 95% CI [3.18, 20.49]).

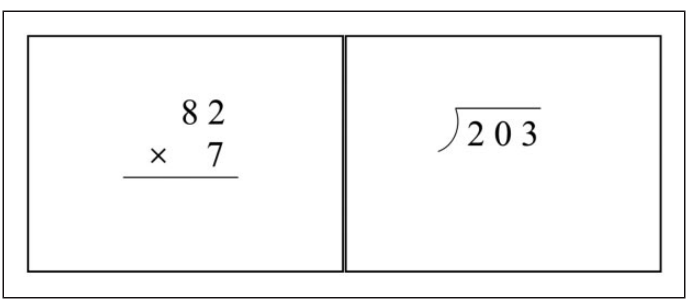

Figure 3. Examples of vertically presented problems (Study 2).

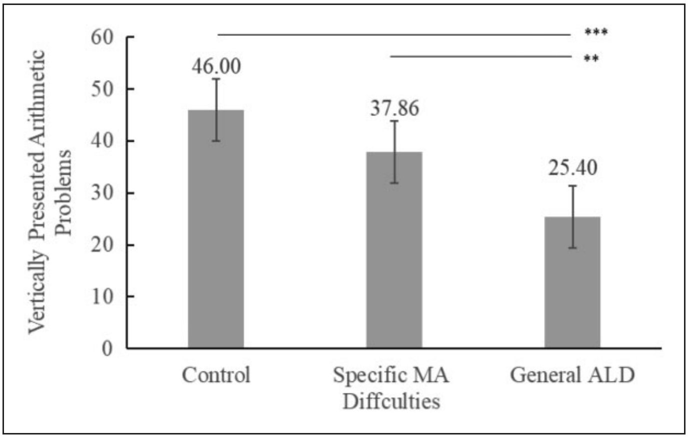

Figure 4. Performance on vertically presented arithmetic problems in three groups (Study 2).
MA: mental arithmetic; ALD: arithmetic learning difficulties.
$^{**}p < .01$, $^{***}p < .001$.

Discussion

In Study 2, we decreased the load on phonological working memory by changing mental arithmetic problems from a horizontal to a vertical format. The findings supported our H2 that the group with specific mental arithmetic difficulties no longer scored significantly below the control group, and the group with general ALD performed significantly worse than the other two groups when problems were vertically presented.

We postulated that vertically presented mental arithmetic problems activated the cognitive processes of calculating by place values that are typically taught in schools through paper-and-pencil format (i.e., participants executed the calculation according to the operant order from ones place to tens place, and so on). The calculation procedure involved in vertical algorithms increased the assistance from visual presentation and decreased the demand on phonological working memory. By alternating problems from horizontal presentation to vertical presentation, we decreased the load on phonological working memory, which was shown to be a weakness in students with specific mental arithmetic difficulties. Thus, they performed better when their phonological working memory load was decreased. However, vertically presented problems did not substantially help students with general ALD to perform as well as the control group because their difficulties in arithmetic might be due to difficulties with arithmetic calculation in general. Adams and Hitch (1997) concluded that if students' poor performance in mental arithmetic is constrained by their poor arithmetic competence rather than their working memory, decreasing working memory load might not help their performance in mental arithmetic.

General discussion

The findings of the present study underscore the importance of phonological working memory in the complex mental arithmetic performance of elementary students. We recruited students with specific mental arithmetic difficulties, those with general ALD, and those without any difficulties in arithmetic. In Study 1, we examined the performance difference on mental arithmetic problems through the perspective of individual differences. Students with specific mental arithmetic difficulties demonstrated substantial difficulties in phonological working memory in comparison to the other two groups. In Study 2, we examined the performance difference on mental arithmetic problems through the perspective of task differences. By changing horizontally presented problems to vertically presented problems (less load on phonological working memory because of the imposed solution procedure that could activate more visual resources), the group with specific mental arithmetic difficulties no longer scored significantly lower than the control group. The present research provides evidence that phonological working memory is involved in mental arithmetic performance, and it might be part of the underlying mechanism contributing to stand-alone difficulties in mental arithmetic.

Educational implications for educators and school practitioners

Some elementary students demonstrate low latencies when solving mental arithmetic problems, and some students with mental arithmetic difficulties also demonstrate difficulties in written arithmetic, much like a double deficit in arithmetic. The findings in Study 1 and Study 2 indicated the importance of differentiating those with specific mental arithmetic difficulties (i.e., a single deficit) from those with general ALD (i.e., a double deficit), and such differentiation has both theoretical and practical implications.

From a theoretical perspective, such differentiation might provide a more precise classification for clinical diagnosis. Students with specific mental arithmetic difficulties might not encounter deficits in basic operation skills. When they do not have access to visual resources (e.g., problems are presented in a format that could activate more visual resources or problems are presented on paper), they have to exhaust more phonological working memory for problem-solving, leading to poor performance. When they regain access to more visual resources, freeing up phonological working memory load, they perform much better in mental arithmetic. However, students with general ALD demonstrate difficulties in both mental and written arithmetic, indicating deficits in basic operation skills. The differentiated classification might help educators to develop targeted interventions to treat the two groups of students differently. For students with specific mental arithmetic, intervention should focus on the process differences between mental arithmetic and written arithmetic (e.g., the role of phonological working memory is different in mental arithmetic versus written arithmetic). They might benefit from strategies that could reduce loads on their auditory working memory, such as providing visual cues (presenting the problems in a format that could utilize more visual resources). For students with general ALD, the intervention should focus on processes that are similar in mental arithmetic and written arithmetic, such as enhancing generic skills of basic arithmetic operation.

From a practical perspective, educators should be aware of differentiated difficulties in students with arithmetic difficulties. For students with general ALD, instructional support and intervention focusing on basic operation skills might help to enhance their performance in mental arithmetic and written arithmetic. However, for students with specific mental arithmetic difficulties, more practice on basic operation skills through pencil-and-paper format might not help (i.e., they might not have deficits in basic operations), and specific training to optimize their phonological working memory might be beneficial.

It is important to note that we classified the participants by using the percentile ranks of their performance in mental arithmetic and written arithmetic in their own classrooms. Such criteria provided relative parameters rather than absolute parameters that could meet clinical standards. That is the reason we applied the term "difficulties" rather than "disorders" in the present study. Students with clinically diagnosed learning disorders often have substantial deficits in a wide range of aspects such as memory, visuospatial orientation, processing speed, and so on (Barnes et al., 2006; McCloskey et al., 1991; Parlato et al., 1992; Rémy et al., 2004), and they might be resistant to instructional intervention utilized in regular education settings. The present study focused only on students with evidence of learning difficulties in mental and written arithmetic, and they might not meet the clinical diagnostic criteria for learning disorders.

Conclusion

Our findings confirmed the important role of phonological working memory in mental arithmetic, concurring with previous studies (Costa et al., 2011; D'Amico & Guarnera, 2005; DeStefano & LeFevre, 2004; Kuang, 2008; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Thevenot et al., 2007; Wang et al., 2006; Zhou et al., 2006). In addition, we found that phonological working memory has a differentiated role in students with specific mental arithmetic difficulties and those with general ALD, suggesting that the underlying mechanism of arithmetic difficulties is different in these two groups.

Among students with specific mental arithmetic difficulties, the findings suggested that (a) their phonological working memory capacity was significantly below the control group (Study 1), and (b) their performance on mental arithmetic tasks was not significantly worse than the control group when problems were vertically presented (more visual resources could be utilized; Study 2). We concluded that (a) students with specific mental arithmetic difficulties have inferior phonological working memory, and (b) large load on phonological working memory during mental arithmetic might be a factor contributing to their poor performance in mental arithmetic; when the working memory load is reduced, they no longer lag behind in mental arithmetic.

Among students with general ALD, we concluded that phonological working memory might not be the core contributing factor for their poor performance in arithmetic. Our findings revealed that (a) they did not demonstrate significantly poorer working memory capacity compared to same-grade typical counterparts (Study 1), and (b) problems presented through vertical presentation (more reliance on visual resources) did not improve their performance of mental arithmetic to the average level, and their performance was significantly worse than the control group. We postulated that their difficulties might be related to their poor basic operation skills that could hinder their calculation performance regardless of whether the arithmetic problems demand large or small load on phonological working memory.

In short, the findings supported the critical role of phonological working memory in mental arithmetic. It is essential to differentiate students with stand-alone difficulties in mental arithmetic and those with difficulties in more than one type of arithmetic problem who might possess a different underlying mechanism for their arithmetic difficulties. Educators are encouraged to differentially treat students' arithmetic difficulties according to their presenting issues and provide instructional intervention that targets their specific difficulties. Students with specific mental arithmetic difficulties might benefit from training that enhances their direct retrieval of arithmetic facts, strategies to maintain intermediate results in mind, and so on. Students could start with problems that are presented in a way that requires a small load on phonological working memory. Through practice and rehearsal, teachers could present increasingly more difficult problems in terms of load on phonological working memory so that students learn to mentally calculate such problems over time. Students with general ALD might benefit from training to enhance their skills in basic operations through paper-and-pencil format. When their basic operation skills are stabilized, strategies to optimize phonological working memory could be introduced.

Limitations and future research

There were a number of limitations of this study. First, we required the participants to enter their answers using computer keyboards during the testing of mental arithmetic. In an ideal setting, mental arithmetic problems often require the participants to provide answers through an auditory modality (i.e., oral responses). Thus, keyboard typing speed might be a confounding variable that could interfere with performance during mental arithmetic. For this reason, we controlled the keyboard typing speed as a covariate to eliminate its impact. Second, we alternated the load of phonological working memory by changing arithmetic problems from a horizontal to a vertical format. The design was based on a previous study indicating that a horizontal format activates more resources on phonological codes, and a vertical format activates more resources on visual aspects (Trbovich & LeFevre, 2003). In the existing literature, there are very few studies that examined the operational approaches of decreasing working memory load through a primary task, which warrants more examination. Third, we focused only on the role of phonological working memory in mental arithmetic in the present study. The underlying mechanism for difficulties in arithmetic could be multifaceted. The difficulties in central executive, visuospatial sketchpad, and other metacognitive aspects in relation to arithmetic warrant further exploration. Fourth, the phonological working memory task was validated in many other empirical studies (Grant & Dagenbach, 2000; Liu et al., 2017; Wang et al., 2008) and that is the reason we adopted this instrument. It is worth to note that the phonological working memory task in this study utilized digits as the stimuli, which might contribute to high correlation with children's arithmetic performance. Phonological working memory tasks utilizing more generic stimuli such as words should be considered in future research. Fifth, since a repeated measures analysis of variance was not conducted, it is possible that maturational processes or skill acquisition could account for the divergent findings across Study 1 and Study 2. Finally, in terms of demographic information, we only collected data regarding the participants' age, gender, and history of documented intellectual disabilities and mental health disorders. Future researchers should consider collecting additional information such as parental education, socioeconomic status, intelligence, verbal reasoning, and vocabulary.

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Author Contributions

YD and R-DL contributed to the design and writing of this study. WH assisted for data analysis and revision of this paper. QY assisted for data analysis and review of literature. JW assisted for computer program development. YL and RZ assisted for data collection and data analysis.

Declaration of Conflicting Interests

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